Young's modulus, density and material properties in cancellous bone over a large density range

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Data on the compressive properties of cancellous bone cubes with a large range of densities (relative densities compared with compact bone of 0.04–0.60) show that the exponent relating the Young's modulus to the density is close to quadratic and that it is improbable that the material Young's modulus of our specimens was less than 8 GPa, and was probably considerably higher.

1. Introduction

The effective matching of prostheses to the bony skeleton requires a detailed knowledge of the mechanical properties of the skeleton. In general, the properties of compact bone are of particular importance. However, cancellous bone may often be on one side of the interface between the prosthesis and the skeleton, and the mechanical properties of cancellous bone are much less well characterized than those of compact bone.

Two questions are currently of great interest in the study of the mechanical properties of cancellous bone: how is the variation in mechanical properties related to variations in density of the tissue, and what are the material properties of the bone making up the trabeculae of cancellous bone? This paper addresses these questions.

In previous studies [1, 2] we examined the effectiveness of two explanatory variables, the apparent density (ρ) and fabric (a measure of the trabecular architecture), in order to explain the compressive Young's modulus (E) of cancellous bone. The first study dealt with non-human bone, and the second with both human and non-human bone. The range of densities in the specimens from the two studies was 94–780 $\mathrm{kg\,m^{-3}}$. This encompasses cancellous bone of extremely low density up to that of moderately high density. (Compact bone has a density of about 1900 $\mathrm{kg\,m^{-3}}$.) In those studies it was possible to account for about 90% of the variance in Young's modulus, using the apparent density and fabric as explanatory variables.

Since then we have carried out further tests on cubes derived from the third metacarpals of horses. These had cancellous bone of relatively high density, ranging from 454 to 1111 $\mathrm{kg\,m^{-3}}$. The addition of these results to the previous data set gave a range of density of an order of magnitude 94–1111 $\mathrm{kg\,m^{-3}}$, rather greater than before, and trebled the value of the greatest Young's modulus observed. This larger data set allows us to determine whether the relationships we previously determined are similar over a greater range of densities. The greater range of densities also allows us to put a firmer lower bound on the Young's modulus of the material of the cancellous bone.

We have not included a discussion of fabric in the present paper. Although the consideration of fabric allows a greater precision in the estimation of the Young's modulus for any cube of cancellous material, for our present purposes the prediction of the Young's modulus is less important than the determination of the effect of density on this property.

2. Materials and methods

The test samples came from the following species and bones: horses 25 (tibia, proximal, 4; metacarpal, 21), bovines, 8 (femur, distal, 5; vertebral centrum, 3); and humans 24 (female, tibia, distal, 3; male, femur, proximal, 5; male, femur, distal, 4; male, tibia, proximal, 5; unknown sex, femur, distal, 7). Other details of the bones, apart from the horse metacarpals, are given in [2].

The testing methods employed are the same as those described more fully in [1, 2]. Cubes of cancellous bone were tested, wet, in a water bath at 37 °C in an Instron 1122 table testing machine, and the Young's modulus of elasticity was determined in each of the three orthogonal directions. For reasons concerned with their use in another experiment, eight of the 21 horse metacarpal cubes were of side length 15 mm, whereas all the others were of side length 10 mm. Specimens were tested at a strain rate varying between about 0.0011 and 0.0033 $\mathrm{s^{-1}}$. (Such differences in strain rate will have negligible effects on the measured Young's modulus [2].)

After mechanical testing, the apparent density was calculated as the dry, fat-free weight of the bone cube divided by the pretesting cube dimensions. Relationships between the variables were determined by conventional regression analysis, using both raw data and

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the data transformed logarithmically. Because the relationships using raw data are strongly curvilinear, we report here results for transformed data only.

3. Results

The results are summarized in Tables I–III and Figs 1–3. We loaded each specimen in three directions, and so have three values of Young's modulus for each cube. These values are themselves highly dependent on the density, and cannot be considered to be independent of each other for any cube. We cannot therefore include them as separate observations in a regression analysis. We report here three stiffness-dependent variables: the mean of the three values of Young's modulus for each cube, and the highest and lowest values for each cube.

The cancellous bone examined came from three evolutionarily widely separated mammalian species: humans, horses and bovines. This range of species was used because it was rather simple thereby to obtain a large range of values of density. It should be emphasized that we are here not concerned with differences in the mechanical behaviour of cancellous bone from different sources, but with the mechanical behaviour of cancellous bone in general. For this purpose the

TABLE I Regression equations using the same slope for the three species, but allowing the intercepts to be different. E_m is the mean Young's modulus, D is apparent density and N is the sample size. In equations such as these the exponent for density is given by the coefficient of log(density)

Mean Young's modulus as a function of density		
		N
1. Horse	$\log E_m = -2.24 + 1.91 \log D$ (0.19) (0.07)	25
2. Human	$\log E_m = -2.30 + 1.91 \log D$ (0.16)	24
3. Bovine	$\log E_m = -2.35 + 1.91 \log D$ (0.18)	8

The R^2-value (97.3%) for the model slope (the proportion of the variance explained by the equation) implies a reduced major-axis (RMA) value of 1.94. Numbers in parentheses are the standard errors of the coefficients.

TABLE II Regression equations for the data split between human and non-human specimens, for mean Young's modulus (E_m) as a function of density

		R^2 (%)	RMA	N
1. Non-human	$\log E_m = -2.32 + 1.93 \log D$	92.0	2.01	33
2. Human	$\log E_m = -2.43 + 1.96 \log D$	94.1	2.02	24
3. All	$\log E_m = -2.45 + 1.97 \log D$	96.8	2.00	57

Predicted values, derived from the whole data set, for mean Young's modulus (MPa) for densities of 100 and 1100 $kg\,m^{-3}$

	100 $kg\,m^{-3}$	1100 $kg\,m^{-3}$
Non-human	33.9	3430
Human	30.9	3390,

TABLE III Highest Young's modulus (E) as a function of density

		R^2 (%)	RMA	N
1. Non-human	$\log E = -2.05 + 1.89 \log D$	77.3	2.15	33
2. Human	$\log E = -1.46 + 1.66 \log D$	85.8	1.79	24
3. All	$\log E = -1.74 + 1.78 \log D$	90.7	1.87	57

Lowest Young's modulus (E) as a function of density

		R^2 (%)	RMA	N
1. Non-human	$\log E = -1.63 + 1.57 \log D$	62.2	1.99	33
2. Human	$\log E = -4.10 + 2.47 \log D$	94.7	2.54	24
3. All	$\log E = -3.52 + 2.24 \log D$	91.6	2.34	57

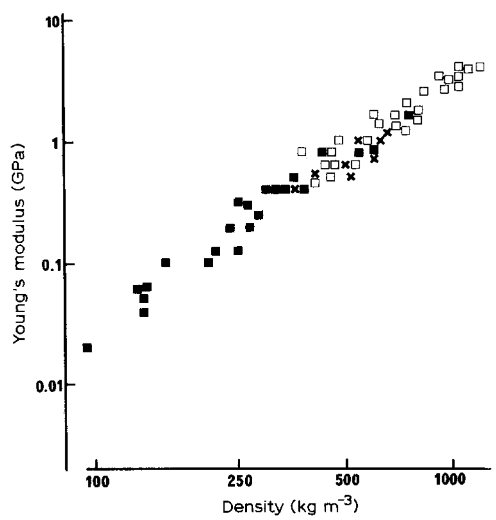

Figure 1 The mean Young's modulus for each cube plotted against density. Note the logarithmic scales. (■) Human, (□) horse and (×) bovine specimens.

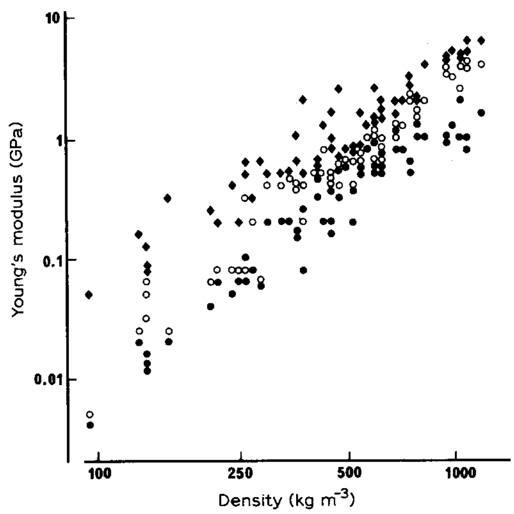

Figure 2 The three values of Young's modulus for each cube plotted against density. Note the logarithmic scales. (◆) Highest value, (○) middle value and (●) lowest value. This diagram shows that the ratio of least to greatest value does not alter much over the range of density.

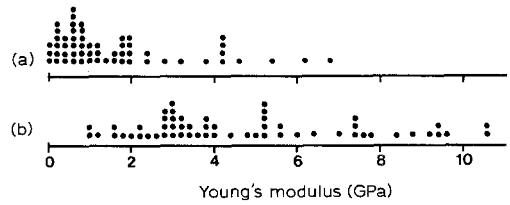

Figure 3 Distribution of the highest Young's modulus of each cube: (a) raw data and (b) data modified to take account of the density of the cubes.

origin of the bone is immaterial, as long as the behaviour of the bone from different species does not differ importantly in respect of the effect of density on the Young's modulus.

It is clear from Fig. 1 that the fit of the data to the general line is so tight that the behaviour of the various species cannot be very different. The relations between the logarithm of Young's modulus and the logarithm of density for the three species were compared using least squares. Candidate models were compared with the maximal model (all lines having different slopes and different intercepts), using the method of multiple comparisons of Aitkin *et al.* [3] Using a confidence level of 80%, only the most restrictive model (single slope) was unacceptable, and that only just so. There was no evidence that the slopes for the three species were different, and accordingly we report in Table I the model in which the lines for all species are parallel but not coincident.

The intercepts of the lines for the different species, which are the logarithms of the values of a in

$$E = a(\text{density})^b$$

are very similar, being -2.24, -2.35 and -2.30 for the horse, bovine and human specimens, respectively. Taking the value of a for horse as 100%, the values for bovine and human are 79% and 87%, respectively. This implies, from the data we have here, that at any particular density the bovine specimens will be 21% less stiff and the human specimens 13% less stiff than the horse specimens. However, our data set includes specimens that differ in stiffness by a factor of 200. Clearly, differences of 20% in the "species" effect, although interesting, are trivial in comparison with the huge differences produced by differences in density, and will not affect the conclusions of this work.

4. Discussion

We discuss the data in relation to the following two points: first, the nature of the power law relating density to the Young's modulus and, secondly, the inferences that can be drawn about the mechanical properties of the bone material of trabecular bone tissue.

4.1. The power-law relationships

There has been much debate concerning the relationship between sample density and the Young's modulus of elasticity. Rice *et al.* [4] comprehensively reanalysed the literature on the subject and concluded that the consensus is that the Young's modulus varies as the square of the apparent density. Carter and Hayes, who pioneered these studies, suggested a cubic relationship [5], and Gibson proposed a mixture of cubic, quadratic and linear relationships according to the range of density being considered [6].

We first discuss the relationship between the mean cube Young's modulus (the mean of the three measures of Young's modulus measured in orthogonal directions in each cube) and density (Fig. 1). The relationship between log(Young's modulus) and log(density) is clearly very close to linear, and therefore a model of the general form

$$\text{Young's modulus} = a(\text{density})^b$$

is appropriate. Density is an extremely effective explanatory variable, accounting for 97% of the variance in mean modulus (Table I). The reduced major-axis value of the density exponent is therefore close to the original exponents in the regression equation, and is extremely close to 2. (When the data are in logarithmic form, as here, the reduced major-axis value is the exponent divided by the correlation coefficient, and is suggested as a more suitable measure of the functional relationship between associated variables by several workers [7].)

We also show, in Table II, the data split between human and non-human data. Splitting the data in this way leaves the values of the coefficients virtually unchanged. It also shows that the regressions for the whole data set are extremely close to the regressions for the species taken individually, so much so that below we in general consider the whole set, referring to species differences only in passing. Table II gives the predicted values for log(Young's modulus) given by the human and non-human regressions at each end of the density range, to show how similar the species predictions are.

Our results suggest that for the total data set, and for the species considered separately, the relationship between the mean Young's modulus and density is considerably closer to quadratic than to cubic.

Of course, the mean Young's modulus is not a material property that is important in any given loading situation; it merely gives a good idea of the general stiffness of the specimen. We therefore also show the results for the greatest measured value of Young's modulus of each cube (which was nearly always the value determined in the proximal–distal direction) and the least value (Table III). There is more scatter in these results, naturally (Fig. 2), but the values for the exponents and the reduced major axes are still close to 2, except for the values for the lowest Young's modulus in human bone.

Gibson [6], using scaling arguments and making assumptions about the relationship between density and the architecture of the bone, demonstrated that there should be a triple relationship between the Young's modulus and density, the exponent being quadratic at lower densities (up to a density of about $350\ \text{kg m}^{-3}$), cubic at higher densities than this and linear at very high densities, at which the structure of the cancellous bone may become a series of parallel-sided tubes, loaded along their length. To test her suggestion we split our whole data set according to Gibson's criterion at a density of $350\,kg\,m^{-3}$ and carried out separate regressions, of mean Young's modulus versus density and of highest Young's modulus versus density. The results are given in Table IV. The exponents give no indication of obeying the theoretical predictions of Gibson, that is of a value of 3 when densities are $> 350\,kg\,m^{-3}$. Indeed, there is a slight but insignificant decrease in exponent value at values of density $> 350\,kg\,m^{-3}$. It is possible that at the very highest values of density this may indicate the third phase of Gibson's three-fold way, although for the distribution for the mean Young's modulus this would not be expected, because the tubes would be loaded along their length in only one of the three directions that are tested. The cubes should be considerably more compliant in the orthogonal loading directions. Moreover, inspection of Fig. 2 shows no tendency for the anisotropy of the Young's modulus to increase with density. Furthermore, there is no trace of Gibson's proposed increase of exponent for densities somewhat below the highest densities (Figs 1 and 2).

4.2. Material Young's modulus

Considerable debate continues over whether cancellous bone material has mechanical properties markedly different from those of cortical bone material. All but the most recent work has been carefully analysed by Rice *et al.* [4], who also produced new estimates of their own. Ryan and Williams [8], using tensile tests on rod-like trabeculae from young bovine femurs, reported a mean value for the Young's modulus of about 1 GPa. Kuhn *et al.* [9, 10] tested small specimens of compact bone and trabeculae of cancellous bone in bending, and found that, although the compact specimens were always stiffer than the cancellous trabeculae, the difference became less as the size of the machined specimens approached the dimensions of the machined trabeculae. The lowest values that they obtained were of the order of 4.5 GPa. Williams and Lewis [11], back-calculating from finite-element models, found that it was necessary to posit a value of 1.3 GPa for the material properties in order to obtain a reasonable fit between theory and experiment. Rice *et al.* [4], back-calculating from the observed Young's modulus of cancellous bone and employing an empirical formula derived by Christensen [12], needed to posit values of 0.61 and 1.17 GPa for human and bovine trabeculae, respectively. They write, however, commonsensically, "These estimates . . . seem to us to be too low".

TABLE IV Regression equations for the whole data set, split at the apparent density of $350\,kg\,m^{-3}$, for mean (E_m) and highest (E) Young's modulus as a function of density

		R^2 (%)	RMA	N
Low density	$\log E_m = -2.84 + 2.14 \log D$	90.1	2.25	16
High density	$\log E_m = -2.20 + 1.88 \log D$	91.5	1.97	41
Low density	$\log E = -2.04 + 1.91 \log D$	78.7	2.15	16
High density	$\log E = -1.89 + 1.83 \log D$	77.2	2.00	41

Some authors have found higher values. Mente and Lewis [13], using a combination of direct testing of irregularly shaped trabeculae and modelling these trabeculae using finite-element analysis, found an average Young's modulus of 7.8 GPa. Townsend *et al.* [14] estimated the Young's modulus from the buckling properties of single trabeculae, and estimated the value to be 11.4 GPa. Ashman and Rho [15] used ultrasonic methods to test the Young's modulus of both the structure and the material of cancellous bone, and found values of 13.0 and 10.9 GPa for human and bovine trabeculae, respectively. Nevertheless, it is a widespread opinion in the literature that cancellous material is considerably more compliant and weaker than cortical bone. The data we have produced allow us to make some estimates of the probable value of the Young's modulus of the cancellous bone material in our specimens.

The point has been made, by Ashman and Rho [15], that the material Young's modulus cannot be less than the measured modulus of a whole specimen. Our denser specimens had quite high values of Young's modulus, despite being very porous. The distribution of Young's modulus for the highest value of each cube is shown in Fig. 3a. Of our 66 specimens, 13 had Young's moduli > 2 GPa, of which eight were > 4 GPa and of which two were > 6 GPa. The highest values, of course, give the best estimates of the material Young's modulus, because lower values are produced both by the low density of the cancellous cubes and by the orientation of the trabeculae being such as to produce large deflections of the trabeculae that do not necessarily reflect large longitudinal strains in the material. Therefore, the highest values, which were found in the dense horse metacarpal cancellous bone, clearly set a lower limit on the possible value for the Young's modulus of the tissue material; this cannot be less than about 5 GPa.

The density of these cubes was considerably less than that of compact bone. If the density of the cancellous bone is D, and the density of compact bone is assumed to be $1900\,kg\,m^{-3}$, then the actual cross-sectional area of bone tissue will be about $D/1900$ of its nominal area. If the value of Young's modulus determined from the deflection of a cube of cancellous bone is E, a minimum estimate of the Young's modulus of the material will, therefore, be $E \times 1900/D$. These values are shown in Fig. 3b. Of the 66 values, eight were > 8 GPa.

Even these values are certain to be lower than the true values of Young's modulus since, because the cancellous bone is so porous, the trabecular struts will be to a large extent loaded in modes such as bending in which the deformations will have no easily determinable relationship to the strains in the material, but will certainly produce overall compressive strain of the cube, which will overestimate the compressive strain in the material. Our results, therefore, although not allowing us to predict the value of Young's modulus of the material, do suggest that this value cannot be less than about 8 GPa, and is indeed certainly higher. Finally, Odgaard and Linde [16] produced evidence that in general values for the Young's modulus of cancellous bone measured in compression are likely to produce underestimates of about 20%, because of the non-uniform distribution of strain in conventionally tested compressive specimens.

The material in our specimens must have Young's moduli greatly in excess of the values suggested by Ryan and Williams [8], Kuhn *et al.* [9, 10], Williams and Lewis [11] and Rice *et al.* [4]. Our results are, however, in agreement with those of our separate study involving the comparison of the mineral volume fraction and microhardness of adjacent cortical and cancellous bone [17]. Values in the trabecular bone were only slightly less than in the cortical bone.

It has been argued (in conversation) that the densest cubes that we studied are not really cancellous bone, but are instead made of cortical bone with many holes in it, and that therefore it is not surprising that, if allowance is made for their holes, they seem to have a rather high modulus. Such semantic arguments are difficult to counter; all we can say is that Fig. 1 shows a great, indeed remarkable, homogeneity in the cancellous bone that we examined over an extremely wide range of densities, and indicates that the only important way in which the cancellous specimens differ is in the amount of bone material present in the cube.

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